Implementation shortfall minimization methodology

Version 1.2

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Revision History

Name	Date	Reason for changes	Version
Peter Gagarinov	7/08/2006	Initial version	0.1
Peter Gagarinov	1/10/2006	Added description of price constraints introduction	0.2
Peter Gagarinov	29/01/2007	Revised overall approach description to include price constraints	0.3
Peter Gagarinov	30/01/2007	Added price risk model description	0.4
Peter Gagarinov	1/02/2007	Added features of practical implementation	1.0
Peter Gagarinov, Andrew Lyubimov	7/02/2007	Fixed several misprints, particularly in transition probabilities/conditional impact forecast formulas.	1.1
Peter Gagarinov	17/09/2007	Fixed typos	1.2

1 Introduction

The paper describes an approach towards calculating optimal order execution schedule based on "implementation shortfall" (ISF) minimization. The methodology proposed uses general impact model for transaction cost estimation and stochastic dynamic programming as a tool for finding optimal execution schedule under constraints on price, market participation ratio and execution horizon. To stay in practical area all the calculations are performed in discrete time using discrete price value partitioning.

2 Basic notations and assumptions

Though many results doesn't rely on assumptions about stock price probability distribution of we nevertheless assume price returns to be normally distributed as it makes a practical application more clear and straightforward.

Assumption 2.1. Assume that observed stock price $S(t)$ follows Geometric Brownian motion discrete process $S_k = S(t_k)$ on time grid $\{t_k\}_{k\in D^k}$, $D^k = \{0, \ldots, N\}$.

We also introduce logarithmic returns $I_k = \ln(S_k/S_{k-1})$ within time $\tau_k = t_k - t_{k-1}, k \in D^k_+$, where $D^k_+ = D^k \setminus \{0\}$, and t_0 - time when order of size X is submitted, $t_N = t_0 + T$ - time by which the order is to be traded completely. We also introduce partition $\mathcal{S}_r = [S^*_{r-1}, S^*_r), r \in D^r = \{1, \ldots, l\}$ consisting of non-intersecting spans called "regimes" that cover whole possible stock price value range $(0, +\infty)$. For each pair of regimes r_{k-1}, r_k observed at times t_{k-1}, t_k denote $\mathcal{I}(r_{k-1}, r_k) = \{\ln(S_k/S_{k-1}) | S_{k-1} \in \mathcal{S}_{r_{k-1}}, S_k \in \mathcal{S}_{r_k}\}$ - set of possible price return values when switching from regime r_{k-1} into regime r_k. And let $\mathcal{F}^r_k(r_{k-1}, r_k) = \sigma\{I_k \in \mathcal{I}(r_{k-1}, r_k)\}$ be information containing the fact that from t_{k-1} till t_k regime changed from r_{k-1} to r_k. Probability distribution of r_k is assumed to be known, namely

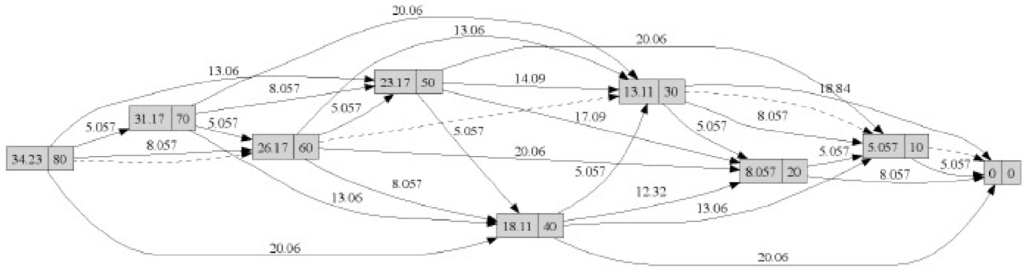

Figure 1: Optimization graph